Primordial Fluctuations in Hybrid Loop Quantum Cosmology

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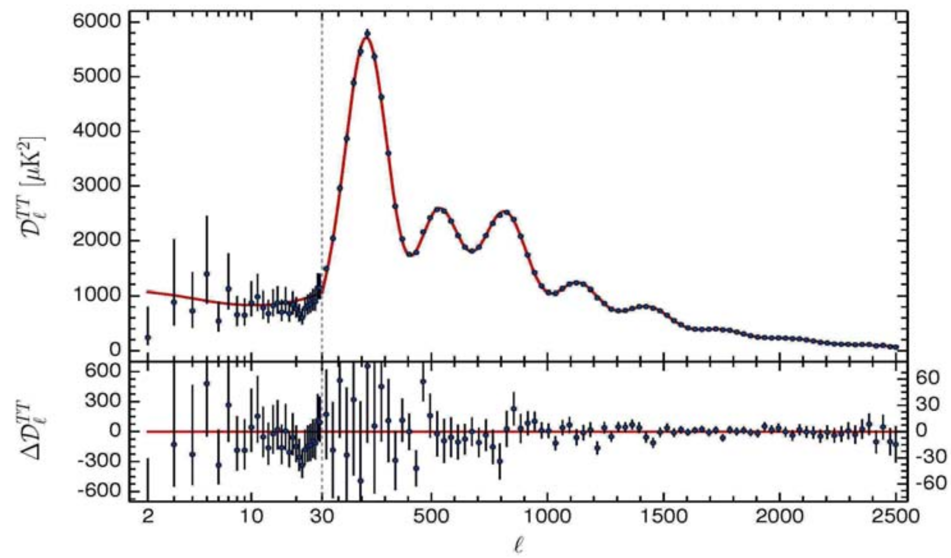
Introduction



- The Universe is approximately homogeneous and isotropic, with cosmological **perturbations**.

- In our era of **precision cosmology**, observational data can be used to falsify models, and potentially even genuine **quantum effects**.

- Collections of observations, including **gravitational waves**, may provide statistical significance.

Model

- We want to study quantum modifications to the equations of primordial fluctuations.

- We consider an **FLRW cosmology** coupled to a **scalar field**.
- For simplicity, we assume **compact flat** (three-torus) spatial topology.
- We focus the dicussion on **TENSOR pertubations**.
- We truncate the action at **quadratic** order in perturbations, with the background treated exactly up to that order.

Classical system

- The FLRW system is described by a **scale factor** and (the zero-mode of) a homogeneous **scalar field:** (a, φ). We set $G = 6\pi^2$.

- We expand the inhomogeneities in transverse traceless **tensor harmonics**.

- Modes are labeled by a non-zero wave vector $\vec{n} \in \mathbb{Z}^3$ and their polarization ϵ. The eigenvalue of the Laplacian is $-\omega_n^2 = -\vec{n}\cdot\vec{n}$.

- **Tensor perturbations** can be described by the coefficients of these harmonics, $T_{\vec{n},\epsilon}$. They are gauge invariants.

- The system as a whole is **symplectic**: zero modes + perturbations.

Gauge invariance

- **Zero-mode** of the Hamiltonian constraint: $H = N_0 \left[H_0 + \sum H_2^{\vec{n},\epsilon} \right]$

N_0: Homogeneous lapse.

$$\boxed{H_0 = \frac{1}{2a^3}\left(-a^2\pi_a^2 + \pi_\varphi^2 + 16\pi^3 a^6 V(\varphi)\right).}$$

$V(\varphi)$: Potential.

- We change the variables for the perturbations to a new canonical set:

☆ The **gauge invariants** $d_{\vec{n},\epsilon} = a\,T_{\vec{n},\epsilon} / \left(32\pi^3\sqrt{3\pi}\right).$

☆ Their **momenta** $\pi_{d_{\vec{n},\epsilon}}$, which are also **gauge invariants**.
A criterion is needed to fix the contribution of $d_{\vec{n},\epsilon}$ to them.

Full system

- We extend the **canonical transformation** to the full system, at the considered **perturbative order**.

$$\tilde{w}_q^a = w_q^a + \frac{1}{2} \sum_{\vec{n},\epsilon} \left[X_q^{\vec{n},\epsilon} \frac{\partial X_p^{\vec{n},\epsilon}}{\partial w_p^a} - \frac{\partial X_q^{\vec{n},\epsilon}}{\partial w_p^a} X_p^{\vec{n},\epsilon} \right].$$

We call $\{w_q^a\} \equiv \{a, \varphi\}$, $\{w_p^a\}$ their momenta, and $\{X_q^{\vec{n},\epsilon}, X_p^{\vec{n},\epsilon}\}$ the old perturbative variables.

- Likewise for $\tilde{w}_p^a$, with a flip of sign in the corrections.

- The corrections are **QUADRATIC** in the perturbations.

New Hamiltonian

- Since the change of zero modes is **quadratic in the perturbations**, the new scalar constraint at our **truncation order** is

$$\boxed{\underbrace{H_0+\sum_b\left(w^b-\tilde{w}^b\right)\frac{\partial H_0}{\partial w^b}+\sum_{\vec{n},\pm} H_2^{\vec{n},\epsilon}}_{\Theta} \quad at \quad \left(\tilde{w}^a,\tilde{X}^{\vec{n},\epsilon}\right).}$$

- Θ is the new perturbative contribution to the constraint.

- It should include **backreaction** at the considered perturbative order.

- This constraint is **quadratic** in the perturbations and independent of $\pi_{\tilde{\varphi}}$ (this may require a redefinition of the *lapse*).

Hybrid quantization

Approximation: Quantum geometry effects are especially relevant in the background.

- Adopt a **(loop) quantum** scheme for zero modes and quantize the perturbations à la **Fock**. The scalar constraint **couples** them.

- A **Fock quantization** is fixed in QFT up to unitary equivalence by the background isometries and the unitarity of the Heisenberg evolution.

- Note that the choice of representation does not fix the **vacuum.**

Hybrid quantization

- Under quantization, physical states must satisfy the **constraint**

$$\boxed{H_S = \frac{1}{2}\left[\pi_{\tilde\varphi}^2 - H_0^{(2)} - \Theta\right],}$$

where $H_0^{(2)} = \tilde a^2 \pi_{\tilde a}^2 - 16\pi^3 \tilde a^6 V(\tilde\varphi)$, $\quad \Theta = \sum_{\vec n,\epsilon} \Theta^{\vec n,\epsilon}$.

$$\boxed{\Theta^{\vec n,\epsilon} = -\left[\left(\vartheta\,\omega_n^2 + \vartheta_M^T\right)(d_{\vec n,\epsilon})^2 + \vartheta\,(\pi_{d_{\vec n,\epsilon}})^2\right],}$$

The same

$$\begin{cases} \vartheta_M^T = \dfrac{H_0^{(2)}}{\tilde a^2} - 32\pi^3 \tilde a^4 V(\tilde\varphi), \\[2ex] \vartheta = \tilde a^2. \end{cases}$$

Both **mode independent.**

Quantization $\longrightarrow \hat H_0^{(2)}.$

Ansatz

- Consider states for which the dependence on the FLRW geometry and the perturbations (N) **split:**

$$\boxed{\Psi = \xi(\tilde{a}, \tilde{\varphi})\,\psi(N, \tilde{\varphi}).}$$

- The FLRW state evolves **CLOSE** to the unperturbed case, with generator $\hat{\tilde{H}}_0$.

- **Approximation**: Disregard transitions to other FLRW states.

Taking expectation values in the **FLRW geometry**, we get a **quantum** constraint for the tensor perturbations:

$$\hat{\pi}_{\tilde{\varphi}}^2 \psi + 2 \langle \hat{\tilde{H}}_0 \rangle_\xi \hat{\pi}_{\tilde{\varphi}} \psi = \langle \hat{\Theta} \rangle_\xi \psi.$$

Propagation equations

- **ASSUMING** a direct effective dynamics for the inhomogeneities, we get the **modified** equations:

$$d^{2}_{\eta_\xi} d_{\vec{n},\epsilon} = -d_{\vec{n},\epsilon}\left[\omega_n^2 + \frac{\langle \hat{\vartheta}^T_M \rangle_\xi}{\langle \hat{\vartheta} \rangle_\xi}\right].$$

Conformal time: $\langle \hat{\widetilde{H}}_0 \rangle_\xi \, d\eta_\xi = \langle \hat{\vartheta} \rangle_\xi \, d\tilde{\varphi}.$ Recall that $\vartheta = \tilde{a}^2.$

- The expectation values give the **quantum corrected mass,** which is **mode independent**.

- The effective equations are **hyperbolic in the ultraviolet** regime.

Example: LQC

- With the **standard variables** (v, b), where $|v| = (2\pi\tilde{a})^3$, and with $\mathrm{V} = 3(2\pi)^3 \gamma \sqrt{\Delta}|v|/2$,

$$\hat{\hat{H}}_0^2 \approx \hat{H}_0^{(2)} = \frac{1}{(2\pi)^3}\left(\frac{\hat{\Omega}_0^2}{(2\pi)^3} - 2\hat{\mathrm{V}}^2 \;\; V\right), \qquad \hat{\Omega}_0 = \frac{1}{2\gamma\sqrt{\Delta}}\hat{\mathrm{V}}^{1/2}\left[\widehat{sgn(v)}\,\widehat{\sin(b)} + \widehat{\sin(b)}\,\widehat{sgn(v)}\right]\hat{\mathrm{V}}^{1/2},$$

Neglecting backreaction

$(2\pi)^3\tilde{a}\,\pi_{\tilde{a}}$

Area gap

Immirzi parameter

$$\Rightarrow \quad \boxed{\hat{\vartheta}_M^T = 4\pi^2\left[\widehat{\frac{1}{\mathrm{V}}}\right]^{1/3}\hat{H}_0^{(2)}\left[\widehat{\frac{1}{\mathrm{V}}}\right]^{1/3} - \frac{\hat{\mathrm{V}}^{4/3}}{2\pi}V, \qquad \hat{\vartheta} = \frac{\hat{\mathrm{V}}^{2/3}}{(2\pi)^2}, \quad \leftarrow \quad \tilde{a}^2.}$$

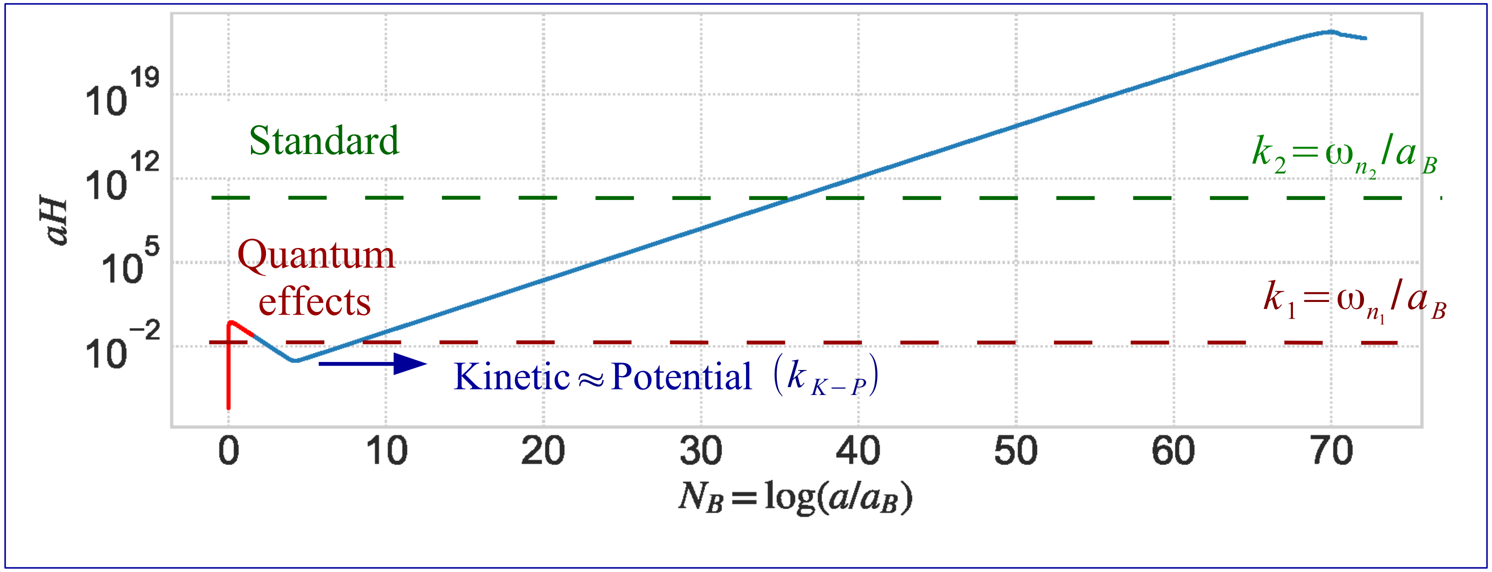
Example: LQC

<u>Possible strategies</u>:

- Compute the quantum expectation values **numerically**.
- Use an **interaction** picture around the massless or the de Sitter case.
- For suitable states, one often adopts the **effective LQC** description.

Initial conditions

- Initial conditions on the *background* within effective LQC:
- Quantum effects affect modes between the scale of LQC and k_{K-P}.
- The effects may be **relevant** and compatible with observations if those modes are entering the Hubble horizon today.



Initial conditions

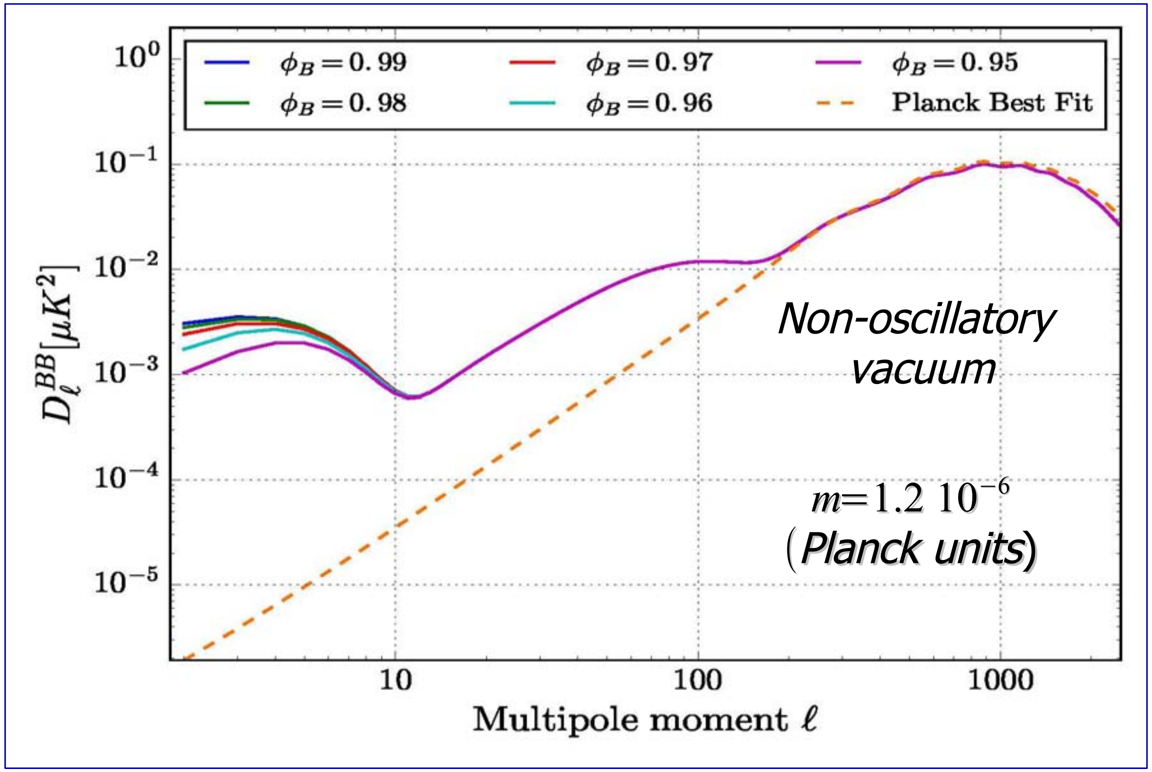
- For backgrounds where this happens, one gets **short-lived inflation**.

- Modes affected by quantum effects:

  - Do **not** leave the Hubble horizon in **slow-roll** regime.

  - Are not in a Bunch-Davies vacuum.

- **Vacuum state**: There are several proposals.



Conclusions

- We have studied tensor perturbations at **quadratic** order in the action.

- At this truncation order, we have found a canonical formulation for the **full system**.

- In a **hybrid quantization**, we have derived propagation **equations** for the perturbations, modified with **quantum corrections** (beyond homogeneous effective descriptions).

- In order to extract predictions, it is essential to determine the **initial conditions** for the background and the vacuum of the perturbations.